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A TRANSISTOR RESONANT VOLTAGE INVERTER WITH PULSE DENSITY MODULATION FOR INDUCTION HEATING EQUIPMENT

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Using a pulse density modulation (PDM) for regulating and stabilization of a transistor converter's load current for induction heating equipment, with the number of pulses during a modulation period divisible to the half-period of an inverter's output voltage is considered in this paper. A mathematical model of the resonant voltage inverter under PDM and analytical equations for maximum and minimum current amplitudes and their difference determining the swing of the ripple current's amplitude were obtained. It is made clear that PDM with the number of pulses during a modulation period which is divisible to the half-period of the inverter output voltage allows, on average, to reduce the swing of the ripple current's amplitude and enlarge the regulation range of the load current. References 6, figures 4.

Key words: transistor resonant voltage inverter, induction heating equipment, pulse density modulation, PDM.

Introduction. Induction heating is widely used in various fields of industry for providing melting, soldering, tempering, heating parts, etc. Transistor converters for high-frequency induction heating systems have a lot of advantages and successfully replace lamps and thyristor generators. Their advantages include high efficiency, small size and weight, longer life time, operation convenience.

For induction heating equipment in the frequency range of 10...440 kHz transistor converters based on voltage resonant inverter with the PDM are widely used [2,3]. The structure of these converters has simple power section, which includes uncontrolled rectifier, a filter, the bridge type's voltage resonant inverter, a matching transformer and a capacitor C. An equivalent load circuit of the converter can be represented as a series connected inductive element L and a resistor R [1].

For providing different technological modes in induction heating equipment it is often necessary to regulate and stabilize the output current or power. Small dynamic power loss which are a significant advantage of inverters with PDM are obtained by switching transistors at a near-zero current that corresponds to the operating mode at a frequency near to the resonant one. For providing this it is being used an automatic frequency control (AFC) [4,5].

Fig. 1 shows the output voltage u and current i of a matching transformer, as well as the control voltage u_s under PDM. It is convenient to evaluate a repetition period T_M of the PDM and on-state time t_{ON} or off-state time t_{OFF} with the aim of integer numbers that are divisible to the output voltage period T_0 of the inverter

$$T_M = sT_0, \qquad t_{ON} = mT_0, \qquad t_{OFF} = nT_0, \tag{1}$$

where n=s-m – the number of periods T_0 during the off-state time t_{OFF}; m – the number of periods T_0 during the on-state time t_{ON}; s – the number of periods T_0 for the period T_M; in other words, numbers n, m, s can be considered as PDM parameters.

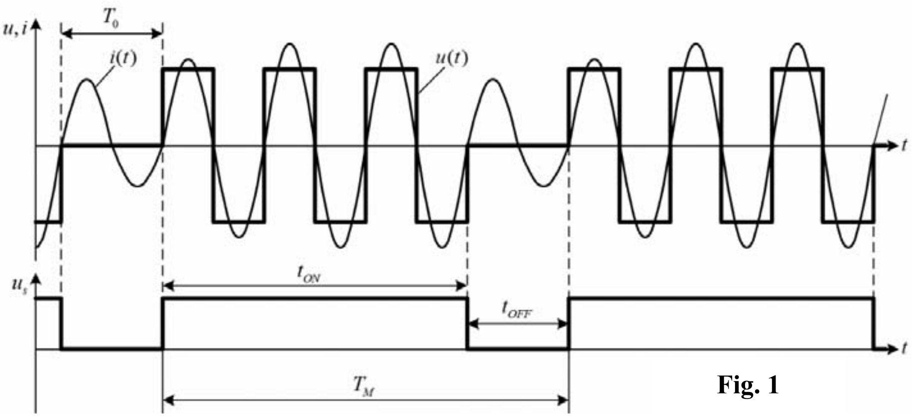

Fig. 1

The main disadvantage of using the PDM to regulate the load current is the swing of the ripple current's amplitude. It influences on the maximum voltage of the resonant circuit's capacitor, on the regulation range of the load current, dimensions of the input filter, power loss in power transistors of the inverter due to AFC function under the PDM and, more generally, on the ability to use the PDM at a low Q factor of the resonant circuit [2,4-6].

The purpose of this paper is to research PDM under the number of pulses during a modulation period divisible to the half-period of the inverter output voltage to reduce the swing of the ripple current's amplitude.

The basic material. Let's consider the PDM with the number of pulses during a modulation period divisible to the half-period of the inverter output voltage. In this case, the number of periods n during t_{OFF} and the number of periods m during t_{ON} will be determined by numbers divisible to 0,5. All possible combinations of PDM parameters (n, m, s) can be divided into four cases: 1) n, m – only integer numbers that are divisible to a period of the inverter output voltage ($\{n\}$=0 and $\{m\}$=0, where $\{n\}$ and $\{m\}$ – fractional part of numbers m and n, respectively); 2) n, m – numbers with the fractional part equal to 0,5 ($\{n\}$=0,5 and $\{m\}$=0,5); 3) n – the number with the fractional part equal to 0,5 ($\{n\}$=0,5), m – the integer number ($\{m\}$=0); 4) n – the integer number ($\{n\}$=0), m – the number with the fractional part equal to 0,5 ($\{m\}$=0,5).

The first case (fig. 1) is investigated in detail in [3]. In the second case, when $\{n\}$=0,5 and $\{m\}$=0,5, the inverter's output voltage has a direct current component leading to saturation of the matching transformer. For the case when $\{n\}$=0,5, m – an integer one two consecutive voltage pulses of the same polarity will be applied to the matching transformer, so the transformer must be designed for the frequency twice lower than the initiative one. This increases the weight and dimensions of the transformer. Therefore, it is inappropriate to use those values of numbers n where $\{n\}$=0,5.

Let's consider in detail the fourth case where $\{n\}$=0, $\{m\}$=0,5 (fig. 2).

Fig. 3 shows equivalent circuits of the serial resonant voltage inverter, matching transformer and resonant circuit for intervals t_{ON} and t_{OFF}, where $u(t)$ is the matching transformer's output voltage.

It is possible to neglect higher harmonics in the inverter's output voltage (because the typical quality factor for induction heating equipment is high) and write the expression for the voltage in the form

$$u(t) = U_{m(1)} \sin\left(2\pi t / T_0\right), \quad (2)$$

where $U_{m(1)}$ is the first harmonic voltage's amplitude at the output contacts of the transformer.

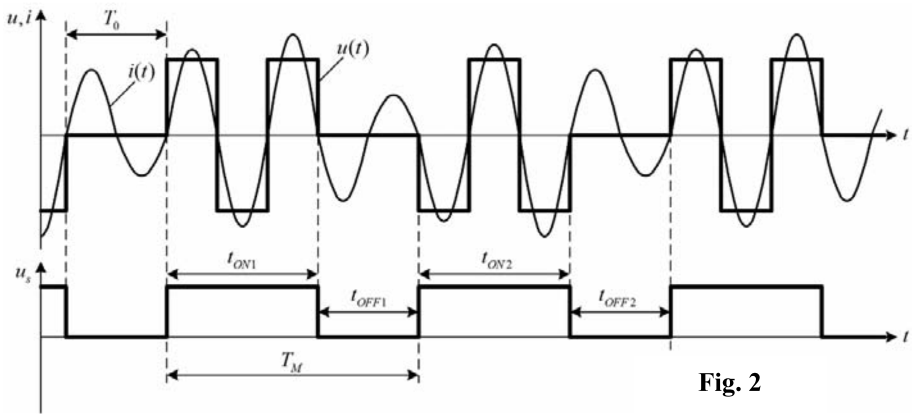

Fig. 2

It significantly helps to simplify analysis and obtain equations in the algebraic form.

In the steady state mode for the adjacent intervals t_{ON1}, t_{OFF1}, t_{ON2}, t_{OFF2} final voltage value across C and current through L on a previous interval coincide with the initial voltage value across C and current through L of the next interval. Applying the Laplace transform's operator method there were defined equations that describe current of the resonant circuit in the case when $\{n\}$=0, $\{m\}$=0,5

$$\begin{cases} i_{ON1}(t) = \dfrac{U_{m(1)}}{R}\sin\left(\dfrac{2\pi}{T_0}t\right) - \dfrac{U_{m(1)}}{R}e^{-\frac{R}{2L}t}\left(\dfrac{1-\exp(-RnT_0/2L)}{1-\exp(-RsT_0/2L)}\right)\sin\left(\dfrac{2\pi}{T_0}t\right); \\ i_{OFF1}(t) = -\dfrac{U_{m(1)}}{R}\dfrac{1-\exp(-RmT_0/2L)}{1-\exp(-RsT_0/2L)}e^{-\frac{R}{2L}t}\sin\left(\dfrac{2\pi}{T_0}t\right); \\ i_{ON2}(t) = -i_{ON1}(t); \\ i_{OFF2}(t) = -i_{OFF1}(t). \end{cases} \quad (3)$$

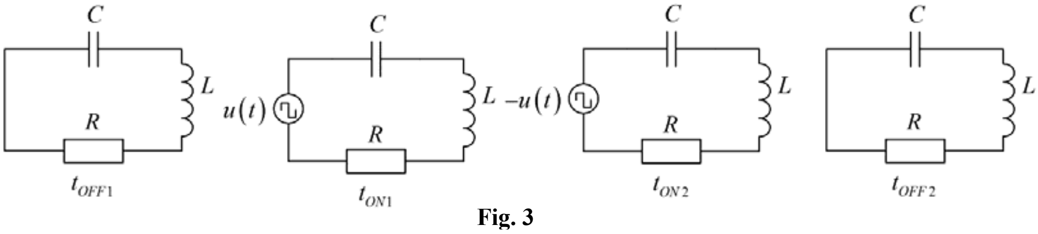

Fig. 3

Taking into account differences between equation for the resonant circuit's current for the case when n, m – only integer numbers [3] and for the case where $\{n\}=0$, $\{m\}=0{,}5$ it is established a generalized mathematical model of the resonant voltage inverter with PDM where the number of pulses during the on-state is divisible to the half-period of the inverter's output voltage and the number of pulses during the off-state time – to a period of the inverter's output voltage

$$\begin{cases} i_{ON1}(t)=\dfrac{U_{m(1)}}{R}\sin\left(\dfrac{2\pi}{T_0}t\right)-\dfrac{U_{m(1)}}{R}e^{-\frac{R}{2L}t}\left(\dfrac{1-\exp(-RnT_0/2L)}{1-\exp(-RsT_0/2L)}\right)\sin\left(\dfrac{2\pi}{T_0}t\right); \\ i_{OFF1}(t)=(-1)^{2m}\dfrac{U_{m(1)}}{R}\dfrac{1-\exp(-RmT_0/2L)}{1-\exp(-RsT_0/2L)}e^{-\frac{R}{2L}t}\sin\left(\dfrac{2\pi}{T_0}t\right); \\ i_{ON2}(t)=(-1)^{2m}\dfrac{U_{m(1)}}{R}\sin\left(\dfrac{2\pi}{T_0}t\right)-\dfrac{U_{m(1)}}{R}e^{-\frac{R}{2L}t}\left(\dfrac{1-\exp(-RnT_0/2L)}{1-\exp(-RsT_0/2L)}\right)\sin\left(\dfrac{2\pi}{T_0}t\right); \\ i_{OFF2}(t)=\dfrac{U_{m(1)}}{R}\dfrac{1-\exp(-RmT_0/2L)}{1-\exp(-RsT_0/2L)}e^{-\frac{R}{2L}t}\sin\left(\dfrac{2\pi}{T_0}t\right). \end{cases} \tag{4}$$

Using the mathematical model (4) there were established analytical expressions defining the minimum $I^*_{a\,\min}$, maximum $I^*_{a\,\max}$ current's amplitudes and the difference between them ΔI^*_a which defines the swing of the ripple current's amplitude in relative units under regulating of an intermediate modulus load's current I_{AVG} (R=const, $I^*_{AVG}=2m/\pi s$=var)

$$I^*_{a\min}(m,s)=\frac{I_{a\min}}{U_{m(1)}/R}=\frac{e^{-\pi n/Q}-e^{-\pi s/Q}}{1-e^{-\pi s/Q}}e^{\frac{\pi}{4Q}}; \tag{5}$$

$$I^*_{a\max}(m,s)=\frac{I_{a\max}}{U_{m(1)}/R}=1-\frac{e^{-\pi m/Q}-e^{-\pi s/Q}}{1-e^{-\pi s/Q}}e^{\frac{\pi}{4Q}}; \tag{6}$$

$$\Delta I^*_a(m,s)=I^*_{a\max}(m,s)-I^*_{a\min}(m,s)=1-\frac{e^{-\pi m/Q}-2e^{-\pi s/Q}+e^{-\pi(s-m)/Q}}{1-e^{-\pi s/Q}}e^{\frac{\pi}{4Q}}. \tag{7}$$

Current's stabilization is an important task both for limiting the current's intermediate modulus value and for providing different technological modes during induction heating. Under load parameters' changes it is also necessary to make current not to fall to zero; otherwise there will occur mal-functions of AFC system. In practice, the load resistance can both increase and decrease during the technological cycle. Also does the inductance L and, respectively, the quality factor Q, although to a lesser extent.

Let's consider the limiting of the maximum current I_{AVG}'s mode which corresponds to the load resistance's decrease from the initial value R_{INI} to the final value R_{FIN}. Under this we assume that the R_{INI} value corresponds with a following γ duty ratio of the modulated voltage: $\gamma=m/s=1$ and L=const. In this case we can write

$$R^*=R/R_{INI}\rightarrow R=R^*R_{INI}, \tag{8}$$

$$Q=Q_{INI}/R^*. \tag{9}$$

Under the stabilization of intermediate modulus' load current it can be obtained from (8):

$$R^*=\frac{2}{\pi}\frac{U_{m(1)}}{R_{INI}I_{AVG}}\frac{m}{s}=\frac{m}{s}. \tag{10}$$

Expressions (5), (6), (7) for the limiting mode of the maximum current I_{AVG} considering with (9) and (10) are given by

$$I^*_{a\min}(m,s,Q_{INI})=\frac{s}{m}\left[\frac{1-e^{-\frac{\pi}{Q_{INI}}\frac{m^2}{s}}}{1-e^{-\frac{\pi}{Q_{INI}}m}}e^{-\frac{\pi}{Q_{INI}}\frac{m}{s}\left(s-m-\frac{1}{4}\right)}\right]; \tag{11}$$

$$I^*_{a\max}\left(m,s,Q_{INI}\right)=\frac{s}{m}\left[1-\left(\frac{1-e^{-\frac{\pi}{Q_{INI}}\frac{m}{s}(s-m)}}{1-e^{-\frac{\pi}{Q_{INI}}m}}\right)e^{-\frac{\pi}{Q_{INI}}\frac{m}{s}\left(m-\frac{1}{4}\right)}\right]; \qquad (12)$$

$$\Delta I^*_{a}\left(m,s,Q_{INI}\right)=\frac{s}{m}\left[1-\left(\frac{e^{-\frac{\pi}{Q_{INI}}\frac{m^2}{s}}-2e^{-\frac{\pi}{Q_{INI}}m}+e^{-\frac{\pi}{Q_{INI}}\frac{m}{s}(s-m)}}{1-e^{-\frac{\pi}{Q_{INI}}m}}\right)e^{\frac{\pi}{Q_{INI}}\frac{m}{s}\left(\frac{1}{4}\right)}\right]. \qquad (13)$$

Using the method of determining PDM parameters [3] allowing to select combinations of PDM parameters for the smaller swing of the ripple current's amplitude, increase its frequency, decrease maximum and increase minimum current amplitudes with taking into consideration a case when the number of pulses during t_{ON} is divisible to the half-period of T_0, there were obtained control characteristics of current I^*_{AVG} and ΔI^*_a in relative units (fig. 4) in the regulation mode of the intermediate modulus' load current under different Q values for two cases: 1) when the number of pulses per time t_{ON} is divisible to the period T_0 (fig. 4 *a*, *c*) and 2) when the number of pulses per time t_{ON} is divisible to the half-period of T_0, (fig. 4 *b*, *d*). Under the calculation it was assumed that $s_{\max}$=10 and limiting condition for current are following: for $I^*_{a\min}$ at the level more than 5% and for $I^*_{a\max}$ – at the level up to 120% of current's amplitude under γ=1.

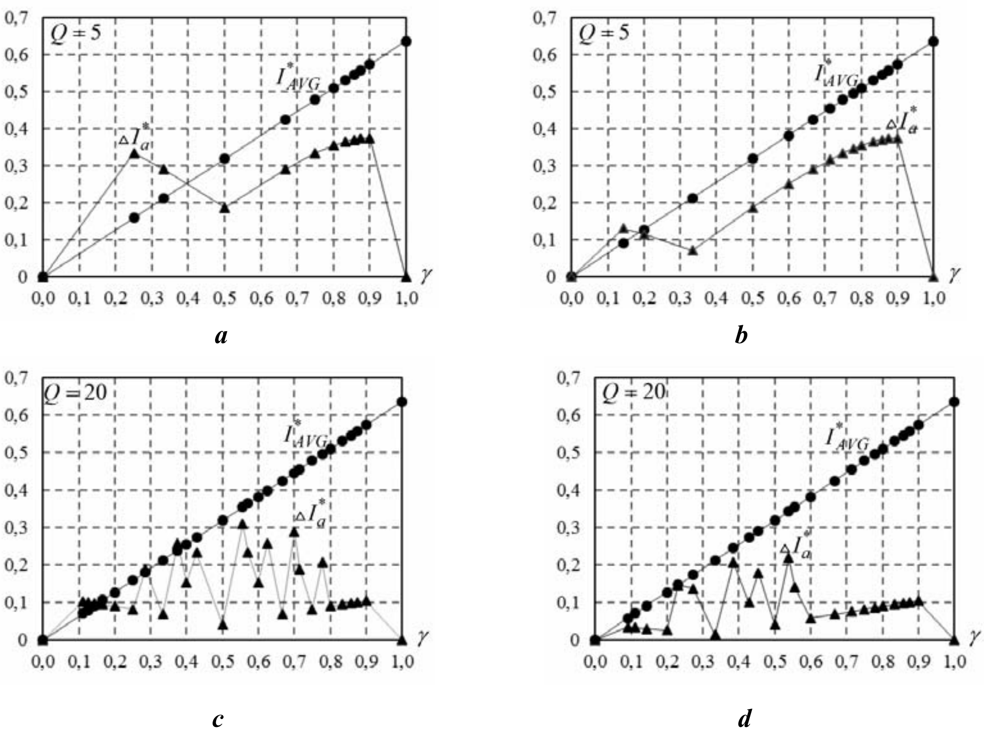

Fig. 4

As shown in fig. 4, using the PDM with the number of pulses during t_{ON} divisible to the half-period of T_0 allows to get more points of control regulation characteristic I^*_{AVG} and, on average, to reduce current ΔI^*_a that, in turn, reduces dynamic power loss during the AFC. When the control system is opened the load current may be changed discretely. Insertion of negative feedback on the regulatory parameters allows to control the load current smoothly throughout the whole range [1]. If the necessary value of γ does not have a corresponding point on the regulation characteristic, then the feedback will provide the value of γ by the aim of combining existing points on this characteristic. However, it is inadmissible to combine the value of γ=0 with other points of regulation characteristic, because for such a case the amplitude $I^*_{a\min}$ will be lower than that given by limit conditions; as a result mal-function of AFC may occur. One more negative effect of combining γ=0 is in reducing the regulation range of current I_{AVG}.

Conclusions. Using the PDM with the number of pulses during the t_{ON} divisible to the half-period of the inverter's output voltage and the number of pulses during t_{OFF} – divisible to the period of the inverter's output voltage, when compared with the PDM when the number of pulses during t_{ON} is divisible to the period of the inverter's output voltage allows, on average, to reduce the swing of the ripple's current amplitude and extend regulation range of the load current. Additionally, dependencies of minimum and maximum current amplitudes and the ripple current amplitude's sweep on PDM parameters (n, m, s) are received.

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ТРАНЗИСТОРНИЙ РЕЗОНАНСНИЙ ІНВЕРТОР НАПРУГИ З НЧ ІМПУЛЬСНОЮ МОДУЛЯЦІЄЮ ДЛЯ УСТАНОВОК ІНДУКЦІЙНОГО НАГРІВУ
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Розглянуто використання НЧ імпульсної модуляції для регулювання та стабілізації струму навантаження транзисторного перетворювача для установок індукційного нагріву, коли кількість імпульсів за період модуляції є кратною півперіоду вихідної напруги інвертора. Отримано математичну модель резонансного інвертора напруги при НЧ імпульсній модуляції та аналітичні вирази максимальної та мінімальної амплітуд струму, а також різниці між ними, яка визначає розмах пульсації амплітуди струму. Встановлено, що використання НЧ імпульсної модуляції, завдяки якій кількість імпульсів за період модуляції кратна півперіоду вихідної напруги інвертора, дозволяє в середньому зменшити розмах пульсації амплітуди струму та розширити діапазон регулювання струму навантаження. Бібл. 6, рис. 4.
Ключові слова: транзисторний резонансний інвертор напруги, установка індукційного нагріву, низькочастотна імпульсна модуляція, НЧ імпульсна модуляція.

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ТРАНЗИСТОРНЫЙ РЕЗОНАНСНЫЙ ИНВЕРТОР НАПРЯЖЕНИЯ С НЧ ИМПУЛЬСНОЙ МОДУЛЯЦИЕЙ ДЛЯ УСТАНОВОК ИНДУКЦИОННОГО НАГРЕВА
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Рассмотрено использование НЧ импульсной модуляции для регулирования и стабилизации тока нагрузки транзисторного преобразователя для установок индукционного нагрева, когда количество импульсов за период модуляции кратно полупериоду выходного напряжения инвертора. Получено математическую модель резонансного инвертора напряжения с НЧ импульсной модуляцией и аналитические выражения максимальной и минимальной амплитуд тока, а также разницы между ними, которая определяет размах пульсации амплитуды тока. Установлено, что использование НЧ импульсной модуляции, благодаря которой количество импульсов за период модуляции кратно полупериоду выходного напряжения инвертора, позволяет в среднем уменьшить размах пульсации амплитуды тока и расширить диапазон регулирования тока нагрузки. Библ. 6, рис. 4.
Ключевые слова: транзисторный резонансный инвертор напряжения, установка индукционного нагрева, низкочастотная импульсная модуляция, НЧ импульсная модуляция.

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