

DOI: <https://doi.org/10.15407/techned2019.01.034>

## STUDY OF STARTING REGIMES OF INDUCTION MOTORS USING EQUIVALENT PARAMETERS OF QUASI-3D FIELD MODEL

O.M. Popovych, I.V. Golovan

Institute of Electrodynamics National Academy of Sciences of Ukraine,  
pr. Peremohy, 56, Kyiv, 03057, Ukraine, e-mail: [popovich1955@ukr.net](mailto:popovich1955@ukr.net)

The efficiency and adequacy of mathematical models of induction motors for dynamic operating conditions are improved due to used dependences for varied electromagnetic parameters. The dependences are based on the results of field analysis. The mathematical models with single-dimensional approximation of parameters are developed. This simplifies the field analysis and formation of approximating dependences with continuous derivatives. The relationship between given functional dependences of electromagnetic parameters and the adequacy of mathematical models for static and dynamic conditions is studied. The expressions and method are proposed to determine the correction coefficients that improve the adequacy of the mathematical models for dynamic conditions with single-dimensional approximation of parameters. References 8, figures 3.

**Key words:** induction motors, parameters of equivalent circuit, main flux reactance, field model, start.

The starting regimes of induction motors (IM) are studied to determine the conformity of their properties to the requirements of the working mechanism, the power line, and also thermal and mechanical stability of IM. There are a number of factors that reduce calculation errors for the dynamic operating conditions. Such of them are the accuracy of determining the magnitude of IM electromagnetic parameters, the adequacy of structure the equations of electrical equilibrium to the features of processes at the demonstration of nonlinear properties of medium. The last is due to the electromotive force (EMF) which is proportional to the speed of inductance change that is – taking into account differential parameters [1]. Besides, the way of representation of nonlinear dependences strongly influences on result of solving a system of differential equations with nonlinear parameters using numerical methods. For example, the application of piecewise-linear approximation leads to a discrete change in the first derivative of the dependence of the change in inductive parameter and to the splashes of the magnitude of differential parameter at the nodes of such nonlinear dependence. There is no smooth approximation. This has a negative effect on the accuracy and stability of the solution of differential equations, especially, in studying and designing of regulated electromechanical systems [2, 3].

The application of field models of induction motors provides the highest accuracy of the study due to a minimal number of simplifying assumptions [4]. But, at the same time, studies need a very great computer time, especially when the dynamic conditions are calculating. This often prevents using the field methods. Therefore, the problem of combination the advantages of field methods by accuracy with circuit methods by speed is actual. This problem has been decided by replacement of quasi3-D field model of IM by equivalent circuit model with two-dimensional nonlinear parameters that are functional dependences on currents and slip [5]. Such an equivalent circuit model provides high accuracy of analysis, but requires the solving a problem of two-dimensional approximation of parameters and correct application in the study of dynamic conditions. The features of the change in the main flux reactance as a function of both changing currents and slip were investigated [6]. The possibility of replacement the two-dimensional approximation by means of single-dimensional is also shown. For this, it's necessary to consider the nonlinear dependences of changes both the saturation factor of the magnetic circuit at the change of total magnetomotive force (MMF) of motor and the change of Carter coefficient at slip change. The efficiency of algorithms for investigating those operating conditions that are associated with a large range of slip changes improves due to the replacement of two-dimensional dependencies of changes in electromagnetic parameters by single-dimensional dependencies. But, the results and methods of applying a single-dimensional approximation of electromagnetic parameters, which are determined using field analysis, require investigation and confirmation.

The purpose of the paper is to compare the results for starting characteristics of squirrel-cage induction motors obtained by field methods with single- and two-dimensional approximation and to develop the propositions for effective models to study the starting regimes of the motors.

A numerical experiment for starting regimes was carried out using the mathematical model of IM of electro-mechanotron system in MATLAB-Simulink [7]. This model provides the possibility of investigating dynamic condi-

tions taking into account the following properties, such as: differential parameters; an arbitrary circuit connecting the branches of the stator winding with each other and with elements of the external circuit; spatial harmonic components of the MMF; losses in steel and additional losses. Two-dimensional dependences of changes in parameters of equivalent circuit for IM (as the function of currents and slip) were determined from results of a series of field calculations. At the same time, the following dependences are used [5]:

$$r'_2 = \frac{P_{ec} \frac{k_l}{k_{cm}} + P_{ek}}{3(I_r^R)^2 + 3(I_r^I)^2}; x_m = \frac{-P_{ec}}{3sI_s I_r^I}; x'_2 = \frac{P_{ek} \operatorname{tg}\varphi_k / 3s}{(I_r^R)^2 + (I_r^I)^2} + x_m \left( \frac{1}{k_{sk}^2} - 1 - \frac{I_r^R I_s k_l}{(I_r^R)^2 + (I_r^I)^2} - k_l \right); x_{ln} = \frac{\omega_0 \Psi_s^R}{\sqrt{2} I_s} - x_m \frac{I_s + I_r^R}{I_s},$$

where, according to the field analysis results:  $P_{ec}, P_{ek}$  are the summary powers of electrical losses in the all rotor rods and its rings;  $I_s, I_r^R, I_r^I$  are the effective values of time complexes of currents for the stator phase of real and imaginary components of the complex of rotor's current leading to the stator;  $\operatorname{tg}\varphi_k$  is the angle tangent of rotor ring current retardation, in comparison with the voltage in the ring;  $\Psi_s^R$  is a real component of full flux linkage complex of the stator phase;  $s$  is the slip;  $k_l = (l_2^2 + h_{sk}^2)/l_1 l_2$  is the rotor slot length change coefficient;  $l_1, l_2, h_{sk}$  are the length of the stator magnetic conductor, the rotor, and the magnitude of slant its slots;  $k_{sk}, k_{cm}$  are the coefficients of the rotor slant slots and filling of magnetic conductor by steel;  $x_{ln}$  is the inductive impedance of stator slot leakage.

Results of study of starting characteristics for IM 4A80A2U3 using the simulation model of IM with two-dimensional parameters [6] (as function of currents and slip) which were obtained using quasi3-D field analysis [5, 7], were compared with studies that based on consider a change in the saturation factor  $k_\mu$  of the magnetic circuit at change of total MMF [8]. The comparison showed that the application of parameters, which are determined by field analysis, reduces the error in determining the value of the initial starting currents and torque by several times. Taking into account the change in the mutual inductance flow path in the field analysis, which crosses the air gap and passes partly through the rotor slot area with the winding, is the reason for this. Herewith increases the equivalent nonmagnetic gap in comparison with the Carter's coefficient  $k_\delta$  constant value [8]. This increase is more than double [6]. It was considered by the variable component of the equivalent air gap coefficient  $k'_\delta$ .

The formation of single-dimensional approximation dependences of electromagnetic parameters as the slip function can be performed based on the values of parameters in the starting and rated conditions:  $x_p, x_n$ , respectively, at slip:  $s_1, s_n$ . We will form the approximating dependences, which will be continuous by derivatives for three ranges:  $s_1 - s_2, s_2 - s_3, s_3 - s_n$ . The parameter changes as the slip function under the linear law, in average range. In extreme ranges, the parameter changes along circles to which the second-range line appears as tangent. The value of derivative equals zero at slips  $s_1, s_n$ . In this case, the value of slip  $s_2$  is determined from solving the next equation:

$$[(x_p - x_n - R - r)(s_1 - s_2)/R]^2 + [(s_1 - s_n)(s_1 - s_2)/R - R - r]^2 - (x_p - x_n - R - r)^2 = 0,$$

where the parameters  $x_p, x_n$  and the corresponding to them radii of circles  $R, r$  are set in relative units of range  $x_p - x_n$ . Other coordinates of the boundaries ranges points are determined by the expressions:

$$s_3 = (s_1 - s_2)r/R + s_n; \quad x_2 = x_p - R + \sqrt{R^2 - (s_1 - s_2)^2}; \quad x_3 = x_n + r - r\sqrt{1 - [(s_1 - s_2)/R]^2},$$

$s$  parameter keeps a constant value outside the ranges of slip change, in limits is determined by the expressions:

$$x(s)_{1-2} = x_p - R + \sqrt{R^2 - (s_1 - s)^2}; x(s)_{2-3} = x_3 + (x_2 - x_3)(s_3 - s)/(s_3 - s_2); x(s)_{3-n} = x_n + r - \sqrt{r^2 - (s_n - s)^2}. \quad (1)$$

The formation of single-dimensional dependence of change in the main flux reactance is performed with consideration the effect of a change in the nonmagnetic gap value and the magnetic circuit saturation [6] in the assumption, that the change of nonmagnetic gap does not affect the values ratio of the falling magnetic potential both in steel of magnetic conductor and the magnetic flux. It is possible to use information about the change in the saturation factor  $k_\mu$  of the magnetic circuit under these conditions, in a function of the summary MMF of motor [8], which is obtained by the method of magnetic circuit sections. The nonmagnetic section can be represented in two parts:  $\delta k_\delta$  and  $\delta k_\delta(k'_\delta - 1)$ , where  $\delta$  is the air gap of the motor;  $k'_\delta = k_{\mu\delta}/k_\mu$  is the accounting coefficient of the change in the nonmagnetic gap by the field analysis results [6];  $k_{\mu\delta}$  is the summary coefficient of change the main flux reactance, according to the field analysis results [6]. The summary coefficient of change in the main flux reactance based on the field analysis results of the steady states of operation, according to the assumptions, studies [6] and the expression of saturation factor for the magnetic circuit  $k_\mu$  as the function of summary MMF of motor [8], we can represent as:

$$k_{\mu\delta}(F_\Sigma, s) = F_\Sigma/F_\delta = 1 + F_{fe}/F_\delta + F'_\delta/F_\delta = k_\mu(F_\Sigma) + k''_\delta(s), \quad (2)$$

where  $F_\Sigma$  is the summary MMF of the motor;  $F_\delta, F_{fe}, F'_\delta$  respectively, are the magnetic potentials falling in the air gap  $\delta k_\delta$ , in the iron, in the replaceable part of nonmagnetic gap  $\delta k_\delta(k'_\delta - 1)$ ;  $k''_\delta(s) = F'_\delta/F_\delta = k_\mu(k'_\delta - 1)$  is the component part of total coefficient of change the main flux reactance, its dependence from slip is determined by the static characteristics of steady state.

The algorithm of determining the dependence  $k''_\delta(s)$  is following: 1) The dependences of the equivalent circuit parameters [5] are determined by the field analysis for both rated and circuited conditions; 2) The values of the equivalent circuit parameters of IM, the values  $k_{\mu\delta}(F_\Sigma, s)$  in the both rated duty and starting regime, and the values  $k_\mu(F_\Sigma)$  for [8] are determined by the circuit analysis of the starting regime and rated duty with the obtained dependences of parameters; 3) These values  $k''_\delta(s) = k_{\mu\delta}(F_\Sigma, s) - k_\mu(F_\Sigma)$  are determined for the chosen points of the static characteristic at slip; 4) Dependence  $k''_\delta(s)$  is approximated by expressions (1).

The calculation results of the steady state for IM 4A80A2U3 operation with two-dimensional parameters as the function of the rotor's rotation frequency (as the function of currents and slip) according to the field analysis results, in particular with given dependence for coefficient of change the main flux reactance  $k_{\mu\delta}(F_\Sigma, s)$  are shown in fig. 1. The values of the currents of regimes calculated by method [8] were determined and the dependence of the saturation factor of the magnetic circle  $k_\mu(F_\Sigma)$  was built. The result of single-dimensional approximation of static dependence for coefficient of change the main flux reactance  $k''_\delta(s) + k_\mu(F_\Sigma)$  is obtained using the approximation of dependence  $k''_\delta(s)$  from expressions (1) with the radius values  $R=0.4$ ,  $r=0.05$ . Fig. 1 shows that both field and classical analysis give the same results starting from working speeds higher  $300 \text{ s}^{-1}$  by the change of main flux reactance.

Results of calculations for static characteristics on two-dimensional parameters of the equivalent circuit by the field analysis almost completely coincide with results of calculation by their single-dimensional approximations, fig. 1. At the same time, the dynamic characteristics at the acceleration of IM with a nominal torque of impedance and with an inertia torque that is 2.5 times larger than the rotor (fig. 2), show, that, at single-dimensional approximation (2), the calculated maxima of the dynamic characteristic of the coefficients of change for main flux reactance are considerably smaller, than at two-dimensional. The exclusion of the influence of electromagnetic transition processes on about half of the main flux reactance that is the result of the establishment in expression (2) dependences  $k''_\delta$  exclusively on slip is the reason of that. It does not influence on result at the static regime analysis, but in the analysis of dynamic conditions needs the correcting.

Additionally, the circuited condition at a voltage, which is reduced by 1.41 times, compared with the rated duty for single-dimensional approximation of parameters taking into account the influence of electromagnetic transitional processes was calculated. In this case, the value of total MMF of the motor and the value of the coefficient  $k''_\delta$  reduction relative to the regime with nominal voltage is determined. According to this information, the expression of a correction coefficient that is equals to one is formed and in the case of MMF value corresponds to a static value for a given slip. The corrected expression of single-dimensional approximation of the coefficient of change the main flux reactance by the field analysis results has the form:

$$k_{\mu\delta}(F_\Sigma, s) = k_\mu(F_\Sigma) + b(F_\Sigma)^a k''_\delta(s), \quad (3)$$

where, for IM 4A80A2U3,  $b = 0.1953$ ;  $a = 0.2615$  (MMF equals 516.3 A in circuited condition at nominal voltage). The corrected dynamic characteristics are given in fig. 3. This correction coefficient doesn't distort static characteristics,

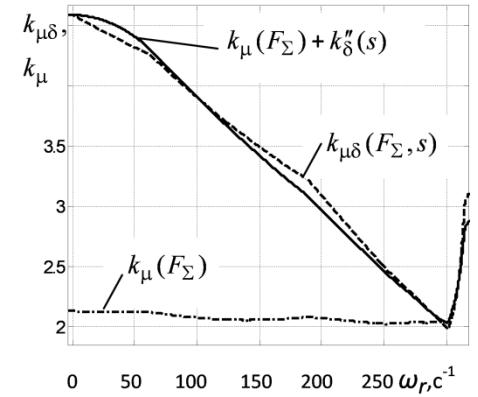


Fig. 1

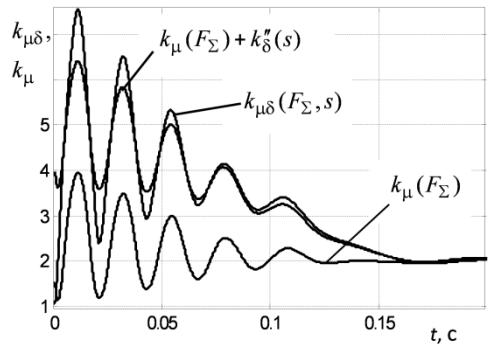


Fig. 2

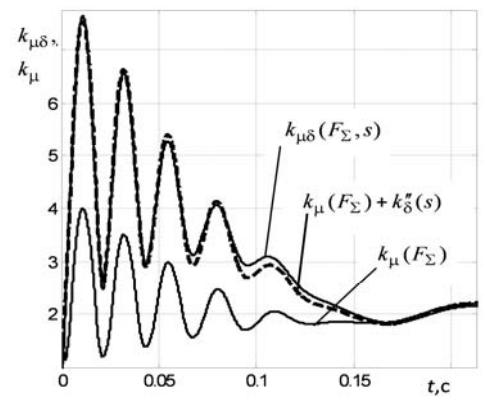


Fig. 3

as in the calculations of regimes with large slip at parameters by field analysis, the calculated value of the total MMF practically doesn't change, and with nominal and higher slip value  $k''_d$  is equal to zero owing to approximation properties (1).

**Conclusion.** The two-dimensional dependences for varied parameters of equivalent circuit of inducting motors as functions of currents and slip obtained by field analysis give a possibility to improve the reliability and accuracy of analysis for starting regimes of the motors when the effective approximating dependences with continuous derivatives are used. The application of such single-dimensional dependences for dynamic condition increases the efficiency of models for starting regimes due to reduced field analysis and simplified determination of approximating dependences with continuous derivatives. The mathematical model of IM with single-dimensional approximating dependences for varied main flux reactance as functions of MMF and slip is developed. The dependences provide high adequacy of the model for static characteristics. The mathematical model of the motors with functional dependence of inductive parameter, or its part, only on slip gives reduced the accuracy for dynamic conditions because the influence of electromagnetic transient processes on the parameters is ignored. The expressions and method for determination of correction coefficients are proposed. For improve the adequacy of the mathematical model for dynamic conditions the model includes single-dimensional approximating dependences for varied electromagnetic parameters as functions of slip.

1. Lesnik V.A., Shurub Yu.V. Account of differential parameters at mathematical simulation of asymmetrical operational modes of asynchronous generators. *Tekhnichna Elektrodynamika*. 2003. No 1. Pp. 45-48. (Rus)
2. Shurub Yu.V. Statistical optimization of voltage regulated induction electric drives. *Tekhnichna Elektrodynamika*. 2014. No 5. Pp. 116-118. (Ukr)
3. Masurenko L.I., Dzhura O.V., Shevchuk S.P. Transients in a Transistor-Switched Capacitor Regulator of a Stand-Alone Induction Generator Supplying a Single-Phase Load. International Conference on modern electrical and energy systems (MEES). Kremenchuk, Ukraine, November 15-17, 2017. Pp. 244-247. DOI: <http://dx.doi.org/10.1109/MEES.2017.8248901>
4. Vaskovskyi Yu.M., Tytko O.I., Makeykin I.S., Kravchuk V.A. Diagnosis of induction motors based on analysis of starting. *Tekhnichna Elektrodynamika*. 2017. No 3. Pp. 58-64. (Ukr)
5. Popovych O.M., Golovan I.V. Refinement of analysis operation of induction motors as part electromechanical systems using equivalent field models using electrical circuits. *Tekhnichna Elektrodynamika*. 2014. No 5. Pp. 113-115. (Ukr)
6. Popovych O.M., Golovan I.V. Study of changed main flux reactance of squirrel-cage induction motors using field analysis of their starting characteristics. *Tekhnichna Elektrodynamika*. 2018. No 5. Pp. 69-72. (Ukr)
7. Bibik O.V., Popovych O.M., Shevchuk S.P. Effective modes of electromechanical system of pump installation of a multi-storey building. *Tekhnichna Elektrodynamika*. 2016. No 5. Pp. 38-45. (Ukr)
8. Radin V.I., Londin Y., Rozenknop V.D. The unified series of asynchronous motors Interelektro. Moskva: Energoatomizdat, 1990. 416 p. (Rus)

УДК 621.313

## ИССЛЕДОВАНИЯ ПУСКОВЫХ РЕЖИМОВ АСИНХРОННЫХ ДВИГАТЕЛЕЙ С ЭКВИВАЛЕНТНЫМИ ПАРАМЕТРАМИ КВАЗИРЕХМЕРНОЙ ПОЛЕВОЙ МОДЕЛИ

А.Н. Попович, докт.техн.наук, И.В. Головань, канд.техн.наук

Інститут електродинаміки НАН України,

пр. Победи, 56, Київ, 03057, Україна. Е-mail: [popovich1955@ukr.net](mailto:popovich1955@ukr.net)

Повышены эффективность и адекватность математических моделей асинхронных двигателей в динамических режимах работы благодаря применению зависимостей изменения электромагнитных параметров по результатам полевого анализа. Разработаны математические модели с одномерной аппроксимацией параметров, что упрощает полевой анализ и формирование аппроксимирующих зависимостей с непрерывностью производных. Исследована связь способа задания функциональной зависимости электромагнитных параметров с адекватностью математических моделей исследования статических и динамических режимов. Разработан способ определения уточняющих коэффициентов для повышения адекватности математических моделей динамических режимов с одномерной аппроксимацией параметров. Библ. 8, рис. 3.

**Ключевые слова:** асинхронный двигатель, параметры схемы замещения, полевая модель, пуск.

УДК 621.313

## ДОСЛДЖЕННЯ ПУСКОВИХ РЕЖИМІВ АСИНХРОННИХ ДВИГУНІВ ЗА ЕКВІВАЛЕНТНИМИ ПАРАМЕТРАМИ КВАЗІТРИВІМІРНОЇ ПОЛЬОВОЇ МОДЕЛІ

О.М. Попович, докт.техн.наук, І.В. Головань, канд.техн.наук

Інститут електродинаміки НАН України,

пр. Перемоги, 56, Київ, 03057, Україна. Е-mail: [popovich1955@ukr.net](mailto:popovich1955@ukr.net)

Підвищено ефективність і адекватність математичних моделей асинхронних двигунів у динамічних режимах роботи завдяки застосуванню залежностей зміни електромагнітних параметрів за результатами польового аналізу. Розроблено математичні моделі з одновимірною апроксимацією параметрів, що спрощує польовий аналіз і формування апроксимаційних залежностей із неперервністю похідних. Досліджено зв'язок способу задавання функціональної залежності електромагнітних параметрів із адекватністю математичних моделей дослідження статичних і динамічних режимів. Розроблено спосіб визначення уточнюючих коефіцієнтів для підвищення адекватності математичних моделей динамічних режимів із одновимірною апроксимацією параметрів. Бібл. 8, рис. 3.

**Ключові слова:** асинхронний двигун, параметри заступної схеми, польова модель, пуск.

Надійшла 05.03.2018  
Остаточний варіант 14.05.2018