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# MULTILEVEL INVERTER TOPOLOGY AND CONTROL SIGNALS DEFINITION BASED ON ORTHOGONAL SPECTRAL TRANSFORMATIONS

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*A method for synthesizing the schemes of multilevel inverters based on the theory of orthogonal transformations of discrete functions is proposed. It is shown that the forming of output voltage of multilevel inverter on the basis of orthogonal transformations of discrete functions defined at finite intervals provides the following advantages: more economical and reliable structure of the power section of the inverter based on unified H-type modules; coefficient of harmonic distortion THD is lower than in known schemes. The generalized order of voltage forming with pulse-amplitude modulation is presented and the choice of the number of inverter modules is justified. The advantages and disadvantages of the received inverter circuits in terms of THD are estimated.* References 6, table 1, figures 3.
***Keywords:*** multilevel inverter, discrete orthogonal transformation, THD, inverter module

**Introduction**. One of the key problems in modern electric power industry is to ensure a high level of energy efficiency of power systems and the quality of the initial parameters of voltage (or current). A special place in modern electrical energy engineering is occupied by power supply systems with distributed generation and renewable energy sources. The output parameter of the main renewable energy sources, such as photovoltaic cells and wind generators is a constant voltage that varies over a wide range. To provide consumers with alternating voltage of 220 V, 50 Hz, the voltage inverters are used. In particular, very actual and complex task is obtaining of the "clean" sinusoid, because even the small distortions of it lead to additional losses and a decrease in the technical and economic performance of the system as a whole. This task is solved by use of multilevel inverters (MLI), which allow obtaining high quality of the output parameter due to the forming of the voltage (or current) with a stepped waveform. The important MLI peculiarity is that the standard low-voltage IGBT modules are used to obtain a high output voltage [5].

Among MLI topologies a specific place is held by cascade multilevel inverters which are based on consecutive connection of several single-phase bridge inverters in a phase of loading [1, 2]. Each single-phase inverter can be considered as an H-type module with a similar topology of power circuits and control. Advantage of such topology is use of the unified modules; high reliability at the expense of reserving capacities; distribution of entrance voltage, current, and power for uniform loading of power switches of the converter. The number of levels of MLI output voltage in this case can be defined as $M = 2s + 1$, where $s$ is the number of modules.

The necessary number of the modules in a phase is defined by required THD (total harmonic distortion) factor. For THD reducing, the number of voltage levels with the *pulse-amplitude modulation* (PAM) should be increased. Actually, for three-level voltage the typical THD value is 25,3%, for five-levels voltage - 11,05% [3].

The objective of the article is elaboration of the ways to achieve assigned level of THD for output voltage (current) of the cascade multilevel inverter with the minimum quantity of inverter modules. This objective is achieved by use of basic functions of spectral transformations as the control signals for the modules and determination of power supply voltage levels for modules or coefficients of transformation of the output (or input) transformer.

**Use of spectral transformations**. The use of spectral transformations for the definition of MLI scheme parameters is based on the mutual orthogonality of the basic functions of their direct and inverse basic functions and the possibility of implementing of these basic functions into inverter modules [4].

The orthogonality of basic functions can be written as

$$\frac{1}{N} \cdot F_r \cdot F_d = \frac{1}{N} \cdot F_d \cdot F_r = I \,, \tag{1}$$

where N is the length of definition interval for initial function and its spectrum (the number of discrete digits), I is a unit matrix with the dimension NxN; $F_r$, $F_d$ are matrices of basic functions of direct and inverse spectral transformations, correspondingly, with the same dimension.

Let the desired voltage with pulse-amplitude modulation is assigned as a column vector U, whose i-th component is equal to the amplitude at i-th tact, i = 0...N-1. The output voltage of MLI will be as follows:

$$U_c = \frac{1}{N} \cdot F_r \cdot F_d \cdot U = F_d \cdot K, \tag{2}$$

where coefficient of transformation K is equal to

$$K = \frac{1}{N} \cdot F_d \cdot U. \tag{3}$$

Equation (2) is the core of MLI output voltage forming. Herewith the rows of $F_r$ are implemented by the voltages of the inverter modules, the elements of the vector $K$ correspond to modules' source voltages (or transformation coefficients), and the multiplication operation of the matrix and the vector corresponds to the addition of the voltages of the modules in the general circuit (or currents in the common node).

Because of mutual orthogonality of direct and return basic functions, the expression (2) will take a form

$$U_c = U, \tag{4}$$

that means that synthesized voltage corresponds to the assigned one.

It's important to note that neglecting of vector $K$ components with small values, the equality (4) will be approximate. But in terms of scheme topology it will allow reducing the number of inverter modules.

To determine the number of modules finally, it is recommended to equate vector's components to zero step-by-step starting from the smallest value. After zeroing of each component THD should be calculated. If THD level is acceptable, the simplification process should stop. The necessary quantity of modules will be equal to the quantity of non-zero components of the vector $K$. The value of each component defines the transformation coefficient, and the corresponding row of $F_r$ defines the number of inverse basic function implemented into the corresponding inverter module.

Among the existing spectral transformations whose basic functions are defined at finite intervals, Walsh transformation could be considered firstly because of its very simple basic functions that accept only the values 1 and -1. While implementing these functions into MLI [6], each invertor modules realizes one of Walsh functions. Resulting THD value is quite high. E.g., for the quantity of invertor modules equal to 3, THD value is 26% [6] that doesn't conform to requirements of modern standards.

Basic functions of Hartley transformation for the interval $N = 3$, spectral transformation at oriented basis (OB) for $N = 3^n$, and the generalized OB transformation for $N = 3^{n1}2^{n2}$ take only values +1, -1, 0 and thus, can be easily implemented into the bridge invertor modules. The advantages of inverter schemes based on the spectral transformations, whose basic functions are defined at the intervals, divisible by 3, are also the simplicity and accuracy of realization of phase shifts divisible by $2\pi/3$, in schemes of three-phase inverters.

When using basic functions of Hartley transformation under the same conditions as Walsh functions (the quantity of inverter modules is 3), the value of THD decreases to 22% [7]. The disadvantage of this method is the impossibility of increasing the number of modules to further reduce THD. This is because Hartley transformation is defined at the interval $N = m^1$, where m is a positive integer, and only for m=3 the basic functions take simple values and are suitable for the implementation into inverter modules. Use of OB and generalized OB transformations [7] requires limitation by a certain number of members in a series. However, in [7] only simplified formula is given for definition of this number. It is necessary to make a reasonable decision about the transformation choice and number of members in discrete series that corresponds to the quantity of inverter modules.

**Generalized method of pam voltage forming**. For modulating function, which is symmetric about T/4 on the positive half-period, the following forming algorithm of PAM voltage could be proposed:
1) Approximation of time modulating function (in the majority a case – sinusoidal function) with the unitary amplitude by a stepped function at the interval T/4; 2) Decomposition of the stepped voltage into an orthogonal series of the favorites of the selected direct transformation at the interval $N = T/4 : T_0$, where $T_0$ is quantization step of sine voltage period; 3) Forming of stepped voltage by the formula of inverse transformation and THD determining; 4) Removal of inverse transformations components with small weight and THD determining by simplified expression; 5) Determining the final inverse transformation formula when sufficient THD level is provided; 6) Defining of MLI structural scheme which implements the basic functions of the inverse transformation into inverter modules, and the values of the spectral components – into the amplitude of modules' supply voltage or the transformation coefficients (Fig. 1 − MLI topology based on unified inverter H-modules).

The method described realizes the forming of multilevel voltage at the interval T/4. To form the voltage throughout the period, the control system is used which generates the control signal for *i*-th module formed as follows:

$$u_{control} = \begin{cases} \varphi_r(i,t), 0 \geq t > \frac{T}{4} \\ \varphi_r\left(i, \frac{T}{4} - t\right), \frac{T}{4} \geq t > \frac{T}{2} \\ -\varphi_r\left(i, t - \frac{T}{2}\right), \frac{T}{2} \geq t > \frac{3T}{4} \\ -\varphi_r\left(i, \frac{3T}{4} - t\right), \frac{3T}{4} \geq t > T \end{cases} \tag{5}$$

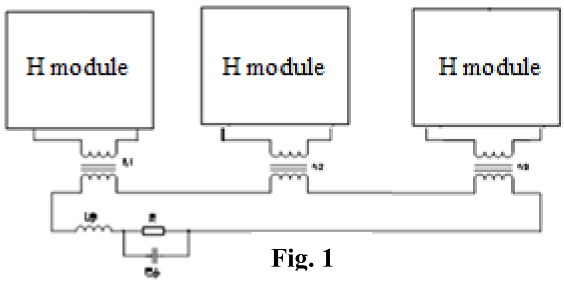

**Fig. 1**

Note that forming of stepped current in inverter modules is carried out in a similar way but instead the addition of voltages in a common contour (see Fig.1) the addition of currents in a common node of inverter modules should be done.

*Spectral OB transformation at N=9.* Let's consider the vector of MLI output voltage as:

$U = (0.044 \quad 0.216 \quad 0.383 \quad 0.537 \quad 0.676 \quad 0.793 \quad 0.887 \quad 0.954 \quad 0.991)^T$.

Each $i$-th component of the vector U corresponds to the value of sine function at $t_i = \left(\frac{iT}{4N}\right) + \frac{T}{8N}$, where T is the period, i=0, 1, 2, N-1.

The matrices of direct and inverse OB transformation are following [4]:

$$F_d = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 & -2 & 1 & 1 & -2 & 1 \\ 1 & 1 & -2 & 1 & 1 & -2 & 1 & 1 & -2 \\ 1 & 1 & 1 & -2 & -2 & -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & -2 & 1 & 1 & 1 & 1 & -2 \\ 1 & 1 & -2 & -2 & 1 & 1 & 1 & -2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -2 & -2 & -2 \\ 1 & -2 & 1 & 1 & 1 & -2 & -2 & 1 & 1 \\ 1 & 1 & -2 & 1 & -2 & 1 & -2 & 1 & 1 \end{pmatrix} F_r = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 0 & 1 & -1 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 1 & 0 & -1 & 1 & 0 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 & 1 & 0 & 1 & -1 \\ 1 & 0 & -1 & -1 & 1 & 0 & 0 & -1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 & -1 & -1 & -1 \\ 1 & -1 & 0 & 0 & 1 & -1 & -1 & 0 & 1 \\ 1 & 0 & -1 & 0 & -1 & 1 & -1 & 1 & 0 \end{pmatrix}. \quad (6)$$

The coefficients of transformation coincide with the spectral components of the function (5):

$$K = \frac{1}{N} \cdot Fd \cdot U = (0.609 \quad -0.006 \quad -0.113 \quad -0.06 \quad 0.027 \quad -0.016 \quad -0.335 \quad -0.023 \quad -0.039)^T \quad (7)$$

Fig. 2 illustrates PAM function at the interval T/4 and its spectrum: *a* – pulse-modulated function, *b* – its spectrum.

The input voltage is described by formula:

$$Uc = Fr \cdot K. \quad (8)$$

This voltage could be written in polynomial form:

$$\begin{gathered} U_c(t) = 0{,}609\varphi_r(0,t) - 0.006\,\varphi_r(1,t) - 0{,}113\varphi_r(2,t) - 0{,}06\varphi_r(3,t) + 0{,}027\varphi_r(4,t) - \\ -0{,}016\varphi_r(5,t) - 0{,}0335\varphi_r(6,t) - 0{,}023\varphi_r(7,t) - 0{,}039\varphi_r(8,t) \end{gathered} \quad (9)$$

where $\varphi_r(i,t)$ is the function of inverse OB transformation that corresponds to $i$-th row in the matrix $F_r$ and should be implemented into $i$-th inverter module.

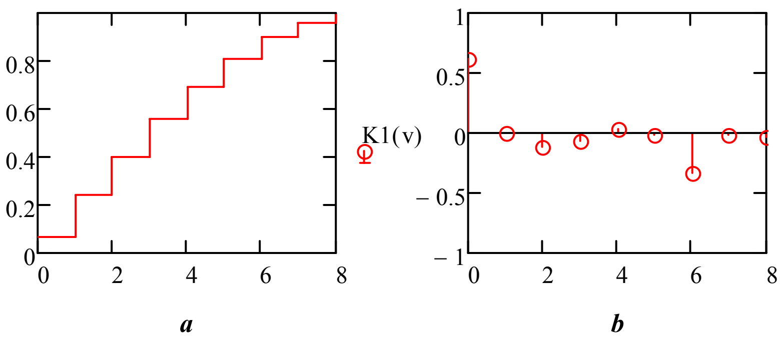


**Fig. 2**

The forming of stepped-waveform $U_c$ is carried out by addition of modules' voltages that correspond to basic functions of inverse transformation with the derived coefficients K.

Table illustrates THD values for different spectral transformations while neglecting the least polynomial members. The data for Walsh functions could be found in [6], for Hartley – in [7]. For OB transformation the values for the voltage were derived by formula (9) with different quantity of additive components. For nine inverter modules THD is 1,215%, for three modules – 6,9%. Similar calculation should be done for the generalized OB at N=6.

It could be seen from Table (THD values for different spectral transformations) that the best quality parameters are achieved while PAM voltage is formed on the base of OB transformation.

| Quantity of inverter modules | THD , % for different transformations | | | |
|---|---|---|---|---|
| | Walsh | Hartley | OB | Generalized OB |
| 3 | 26 | 22 | 6.9 | 15,2 |
| 4 | 10 | | 4 | 8,6 |
| 5 | | | 3,9 | 5 |
| 6 | | | 2.6 | 4,3 |
| 9 | | | 1.215 | |

**Conclusions.**

Forming of the output voltage of multilevel inverter on the base of orthogonal spectral transformations provides low level of harmonic distortion of the output voltage.

The least THD value (1,215%) was received for spectral transformation at oriented basis (OB) with 9 inverter modules. However, in terms of more economical effectiveness it's recommended to use 3 or 4 inverter modules that are able to provide THD equal to 6,9 and 4% correspondingly.


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**СИНТЕЗ СХЕМ МНОГОУРОВНЕВЫХ ИНВЕРТОРОВ НА БАЗЕ ОРТОГОНАЛЬНЫХ СПЕКТРАЛЬНЫХ ПРЕОБРАЗОВАНИЙ НА КОНЕЧНЫХ ИНТЕРВАЛАХ**

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*Предложен способ синтеза схем многоуровневых инверторов на базе теории спектральных преобразований дискретных функций. Показано, что формирование выходного напряжения многоуровневого инвертора на базе ортогональных преобразований функций, определенных на конечных интервалах, обеспечивает следующие преимущества: экономичную и надежную структуру силовой части инвертора на основе унифицированных модулей Н-типа и меньший по сравнению с известными схемами коэффициент гармонических искажений THD. Приведены обобщенный порядок синтеза напряжения с амплитудно-импульсной модуляцией и обоснован выбор количества инверторных модулей. Оценены преимущества и недостатки полученных схем инверторов с точки зрения THD.* Библ. 7, табл. 1, рис. 2.
***Ключевые слова:*** инвертор напряжения, дискретные ортогональные преобразования, инверторный модуль

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**СИНТЕЗ СХЕМ БАГАТОРІВНЕВИХ ІНВЕРТОРІВ НА БАЗІ ОРТОГОНАЛЬНИХ СПЕКТРАЛЬНИХ ПЕРЕТВОРЕНЬ НА СКІНЧЕНИХ ІНТЕРВАЛАХ**

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*Запропоновано спосіб синтезу схем багаторівневих інверторів на базі теорії спектральних перетворень дискретних функцій. Показано, що формування вихідної напруги багаторівневого інвертора на базі ортогональних перетворень функцій, визначених на скінчених інтервалах, забезпечує наступні переваги: економічну і надійну структуру силової частини інвертора на основі уніфікованих модулів Н-типу та менший у порівнянні з відомими схемами коефіцієнт гармонійних спотворень THD. Наведено узагальнений порядок синтезу напруги з амплітудно-імпульсною модуляцією та обґрунтовано вибір кількості інверторних модулів. Оцінені переваги і недоліки отриманих схем інверторів з точки зору THD.* Бібл. 7, табл. 1, рис. 2.
***Ключові слова****:* інвертор напруги, дискретні ортогональні перетворення, інверторний модуль

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