

## DESIGN OF CONTROL SIGNALS FOR THREE PHASE MATRIX CONVERTER ON THE BASIS OF DOUBLE FOURIER SERIES

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*A derivation of control signals of three phase matrix converters is based on extension of the space with one to the space with two time variables. The control signals are presented in form of Fourier series with four coefficients. The procedure is based on equating appropriate coefficients corresponding harmonics of double Fourier series and on solving obtained equations. References 3, figures 1.*

**Key words:** matrix frequency converter, double Fourier series.

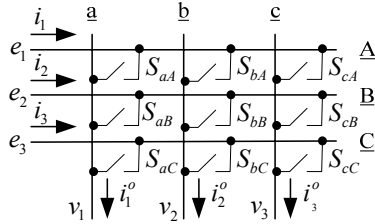
**Introduction.** The processes in the circuits of three phase matrix frequency converters (MFC) are described by the following equations

$$V = M(t)E, \quad I = M^T(t)I^o \quad (1,2)$$

where  $E = \begin{pmatrix} E_i \cos(\omega t) \\ E_i \cos(\omega t + 2\pi/3) \\ E_i \cos(\omega t + 4\pi/3) \end{pmatrix}$ ,  $V = \begin{pmatrix} V_o \cos(\omega_L t) \\ V_o \cos(\omega_L t + 2\pi/3) \\ V_o \cos(\omega_L t + 4\pi/3) \end{pmatrix}$  are vectors of the input and output voltages,

$I = \begin{pmatrix} I_i \cos(\omega t + \phi) \\ I_i \cos(\omega t + 2\pi/3 + \phi) \\ I_i \cos(\omega t + 4\pi/3 + \phi) \end{pmatrix}$  and  $I^o = \begin{pmatrix} I_o \cos(\omega_L t + \psi) \\ I_o \cos(\omega_L t + 2\pi/3 + \psi) \\ I_o \cos(\omega_L t + 4\pi/3 + \psi) \end{pmatrix}$  are vectors of the input and output currents,  $\omega$  and  $\omega_L$  are pulsations of the input and output signals.

The circuit topology of such a converter is shown in Figure.



The switches  $S_{aA} \dots S_{cC}$  are assumed as ideal. They are turned on and off periodically with appropriate time intervals defined by the matrix

$$M(t) = \begin{pmatrix} m_{aA} & m_{aB} & m_{aC} \\ m_{bA} & m_{bB} & m_{bC} \\ m_{cA} & m_{cB} & m_{cC} \end{pmatrix}$$

The theory of MFC is based on a control of input and output voltages and currents [1]. Since a choice of control signals is not univocal [2], we will show what conditions are necessary to derive these signals. In this paper we use substitution [3]

$$t \rightarrow \Omega, \quad \omega \rightarrow \tau, \quad \omega_L \rightarrow \tau \quad (3)$$

in order to find control signals  $m_{nK}$  using the double Fourier series.

**Mathematical model.** Let us assume that coefficients of the matrix  $M(t, \tau)$  are periodic and can be

described as follows 
$$m_{nK} = \sum_{l=0}^4 [ms_{nK}^l \sin(k\Omega\tau) + mc_{nK}^l \cos(k\Omega\tau)] \quad (4)$$

where coefficients  $ms_{nK}^l$ ,  $mc_{nK}^l$  might depend on time  $\tau$ ;  $n = a, b, c$ ;  $K = A, B, C$ .

In order to find these coefficients let us multiply the first row of the matrix  $M(t, \tau)$  by the vector  $E$  and then equate it to the output voltage

$$V_o \cos(\Omega\tau) + V_3 \cos(3\Omega\tau) + E_3 \cos(3\Omega\tau) = m_{aA} E_i \cos(\Omega\tau) + m_{aB} E_i \cos(\Omega\tau + 2\pi/3) + m_{aC} E_i \cos(\Omega\tau + 4\pi/3) \quad (5)$$

The terms  $V_3 \cos(3\Omega\tau)$  and  $E_3 \cos(3\Omega\tau)$  determine a shift of a virtual zero of the three phase system. It makes possible to obtain the gain:  $V_o / E_i = \sqrt{3}/2$  [1]. As it has been shown in [1] voltages  $V_3 = -1/6V_o$  and  $E_3 = \sqrt{3}/6V_o$ .

By using the condition  $\mathbf{1} = M(t, \tau)\mathbf{1}$  (where  $\mathbf{1}$  is the unit vector), we get:  $m_{aC} = 1 - m_{aA} - m_{aB}$ . After substitution of  $m_{nK}$  from (4) in (5) and equating to zero expressions for corresponding harmonics we obtain a system of equations from which we eliminate some of  $ms_{aA}^l$ ,  $mc_{aA}^l$ ,  $ms_{aB}^l$ ,  $mc_{aB}^l$  coefficients. Then we use the same procedure to find  $ms_{bA}^l$ ,  $mc_{bA}^l$ ,  $ms_{bB}^l$ ,  $mc_{bB}^l$  and  $ms_{cA}^l$ ,  $mc_{cA}^l$ ,  $ms_{cB}^l$ ,  $mc_{cB}^l$  coefficients. For input and output voltages we use the matrix equation (1).

Now we use the matrix equation (2) for input and output currents. Let us consider the equation

$$I_i \cos(\Omega\tau) = m_{aA} I_o \cos(\Omega\tau) + m_{bA} I_o \cos(\Omega\tau + 2\pi/3) + m_{cA} I_o \cos(\Omega\tau + 4\pi/3). \quad (6)$$

We again form a system of equations by equating to zero expressions for corresponding harmonics. In this equations the coefficients  $m_{bA}$  and  $m_{cA}$  are obtained from  $m_{aA}$  by changing indexes  $a$  for  $b$  and  $a$  for  $c$  and by substituting  $\Omega\tau$

for  $\Omega\tau + 2\pi/3$  and  $\Omega\tau$  for  $\Omega\tau + 4\pi/3$ . After solving of these equations we find all  $m_{nk}$  coefficients. We also obtain the expression  $V_o I_o = E_i I_i$ , which corresponds to equality of the input and output powers.

It should be noted that in  $m_{nk}$  some of  $ms_{nk}^l$ ,  $mc_{nk}^l$  coefficients are not defined. First, we use the symmetry condition: it means that the steady-state of  $m_{nk}$  should equal 1/3. Second, we consider that amplitudes of harmonics in  $m_{ak}$  are the same. In that manner we again eliminate some coefficients. For the sake of conciseness we show only the coefficient  $m_{a4}$

$$m_{a4} = 1/(9E_i) \left\{ 3E_i + (-9mc_{ca}^4 E_i + 6E_3) \cos(2\Omega t) + 6 \cos(\Omega t) [V_0 \cos(\Omega\tau) + V_3 \cos(3\Omega\tau)] + 2\sqrt{3} \cdot [3mc_{ab} E_i + V_0 \cos(\Omega\tau) + V_3 \cos(3\Omega\tau)] \sin(\Omega t) + 9E_i [mc_{ca}^4 \cos(4\Omega t) + ms_{ca}^4 [-\sin(2\Omega t) + \sin(4\Omega t)]] \right\}.$$

One can see, that we can eliminate some harmonics by simply setting  $ms_{ca}^4 = 0$  and  $mc_{ab} = [V_0 \cos(\Omega\tau) + V_3 \cos(3\Omega\tau)] / (3E_i)$ . Consequently

$$m_{a4} = \frac{\left\{ E_i + (-3mc_{ca}^4 E_i + 2E_3) \cos(2\Omega t) + 3mc_{ca}^4 E_i \cos(4\Omega t) + 2 \cos(\Omega t) [V_0 \cos(\Omega\tau) + V_3 \cos(3\Omega\tau)] \right\}}{3E_i}.$$

Now we choose the coefficient  $mc_{ca}^4$  in order to obey the conditions  $0 \leq m_{n,k} \leq 1$ . Analysing the extremes of  $m_{a4}$  with respects to  $t$ ,  $\tau$  and  $mc_{ca}^4$ , we find that extremes are outside of the intervals  $0 \leq m_{n,k} \leq 1$ . So, we should find extremes with respects to  $t$ ,  $\tau$ , equate  $m_{a4} = 1$  or  $m_{a4} = 0$  and determine  $mc_{ca}^4$ . After substitution the obtained value of  $mc_{ca}^4$  we get

$$m_{a4} = \frac{\left\{ 18E_i + (36E_3 + \sqrt{3}V_0) \cos(2\Omega t) - \sqrt{3}V_0 \cos(4\Omega t) + 36 \cos(\Omega t) [V_0 \cos(\Omega\tau) + V_3 \cos(3\Omega\tau)] \right\}}{54E_i}.$$

It should be noted, that obtained expressions coincide with expressions given in [1] for the case when phase shifts are equal to zero.

**Conclusions.** In this paper control signals for three-phase matrix frequency converter are founded. The method is based on the use of the double Fourier series and the extension of the space of independent time variable. The system of equations for determining of control signals is obtained and solved.

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УДК 621.314

**Моделювання сигналів управління трифазного матричного перетворювача на основі подвійного ряду Фур'є**  
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*Моделювання сигналів управління трифазних матричних перетворювачів базується на розширенні простору з однією у простір з двома змінними часу. Сигнали управління представлені у вигляді ряду Фур'є з чотирма коефіцієнтами. Процедура заснована на порівнянні коефіцієнтів відповідно гармонік подвійного ряду Фур'є і на вирішенні отриманих рівнянь. Бібл. 3, рис. 1.*

**Ключові слова:** матричний перетворювач частоти, подвійний ряд Фур'є

**Моделирование сигналов управления трехфазного матричного преобразователя на основе двойного ряда Фурье**  
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*Моделирование сигналов управления трехфазных матричных преобразователей основано на расширении пространства с одной в пространство с двумя переменными времени. Сигналы управления представлены в виде ряда Фурье с четырьмя коэффициентами. Процедура основана на приравнивании коэффициентов относительно гармоник двойного ряда Фурье и на решении полученных уравнений. Библ. 3, рис. 1.*

**Ключевые слова:** матричный преобразователь частоты, двойной ряд Фурье.

Надійшла 10.02.2014